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Mathematical approach to information and knowledge

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Resume
The article introduced the basic axioms of the mathematical theory of information: from the elementary volume of information to the point of information and information ensemble of any size and complexity. It is shown that the current configuration of the information ensemble correlates with the contents of space, which is configured at the current time. It was present relations for the density of information at any point in the ensemble and noted that under certain assumptions, information density is determined only by the history of the information ensemble. It was written in the principle of conservation of information for open informational ensemble and the expression of the second law of thermodynamics for the transformation of information into energy was obtained too. The assessment of the effectiveness of the information to energy transformation was done and it is shown that «knowledge» in simple physical system — is the part of the information that can be transformed into energy in accordance to the laws of thermodynamics. The probabilistic interpretation of the effectiveness of information to energy transformation was given, which introduces the value of knowledge through the formula for Shannon's information. Submitted relations are fundamental for the understanding and presentation of applied informatics sciences disciplines such us medical informatics.

Key words: information, conservative law, thermodynamics, knowledge.

Клин. информат. и Телемед. 2011. Т.7. Вып.8. с.7–9

Introduction

«Information is information, not matter or energy» ingeniously wrote Norbert Wiener in 1948 [1]. There was need for more than 60 years for real understanding and experimental proofing for such thesis. Professor Masaki Sano from Tokyo University did experimental demonstration of information-to-energy conversion according to thermodynamics law [2]. Thus information takes place among fundamental grounds of the Nature – space, time, energy and matter. At the same time part of the existing axiomatic theory of information requires significant improvement, which we are trying to fill in this article. Moreover, such axiomatic are fundamental for the understanding and presentation of applied informatics sciences disciplines such as medical informatics.

Results

Let's introduce the concept of an elementary volume of information ∂V within which information distribution ∂I is uniform and given by its density ρ[1]:

$$\rho=\frac{\partial I}{\partial V}\ . \qquad (1)$$

Note that the point information δI is a limiting case of an elementary volume of information, when

$$\delta I=\lim_{\partial V\to 0}\rho\partial V\,. \qquad (2)$$

Now we define an information ensemble I as a configuration $\chi(V,t)$ that occupies a point of time t the amount of space V for which

$$I=\int_{\chi(V,t)}\rho dV\,. \qquad (3)$$

It is evident that the configuration $\chi(V,t)$ brings into correlation information I with space region V, which configuration holds at the given moment of time t.[2]

If we can trace the evolution of the initial configuration $\chi(V,t)$ so there is one map λ that takes $\chi_1(V,t)$ in $\chi_2(V,t)$, at a constant amount of information I, then

$$I=\int_{\chi_1(V,t)}\rho_1 dV=\int_{\chi_2(V,t)}\rho_2 dV. \qquad (4)$$

If the mapping λ is continuously differentiable and denotes J as Jacobean:

$$J=\left|\det(\nabla\lambda)\right|, \qquad (5)$$

the using the theorem of integral calculus we get

$$\int_{\chi_1(V,t)}\rho_1 dV=\int_{\chi_2(V,t)}\rho_2 J dV \qquad (6)$$

or

$$\rho_2 J=\rho_1. \qquad (7)$$

Thus, the density of information in a configuration defines the density of all the rest.

Let's make some concluding remarks about introduced concepts. As for the elementary volume of information (according to the representation of uniformity), – we can construct a distributive lattice with the addition, which regulates the

[1]*For example, the information storage industries are common for use technical unit of the recording density of information – bit/in^2.*
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amount of information in the elementary Boolean algebra [3]. Regarding to information ensemble we can assert that it satisfies the fundamental theorem of decomposition, – every ensemble B can be representing in the form of a compound of any of its parts β with to separate ensemble A:

$$(B = A \vee \beta)\,\&\,(A \wedge \beta = 0) \tag{8}$$

here, ∨ – denotes connection; ∧ – superposition. Now, in addition, we can note that for the chosen sequence of nested pieces of information i_k of the information ensemble I (when all i_k have exactly one common point and $V(i_k)$ tends to zero as $k \to \infty$) information density is given by (at almost every point x of the information ensemble):

$$\rho_i(x,t) = \lim_{k\to\infty} \frac{i_k}{V(i_k)} \,. \tag{9}$$

There are enough for further statement of arguments.

Let us consider an informational open ensemble (i.e. possible exchange of information) for which the principle of conservation of information can be written in the form of an integral balance equation:

$$\frac{d}{dt}\int_{\chi(V,t)} \rho dV = \int_{\chi(V,t)} \dot{\rho} dV = \int_{\partial\chi(V,t)} \Phi n dA + \int_{\chi(V,t)} \rho \Psi dV. \tag{10}$$

Thus, the balance equation postulates that the rate of change of information in any part of the configuration $\chi(V,t)$ of the information ensemble represented as the sum: information flow across the configuration border $\partial\chi(V,t)$ and production of information within the configuration. The value Φ – defines information flow across the border and Ψ – inner information production. The above balance equation is valid for all information ensembles and for all parts of their configurations.

We can move to the differential form of the balance equation (10) by imposing additional conditions of configuration boundary smoothing and continuity for the information flow Φ[3]:

$$\int_{\chi(V,t)} (\dot{\rho} - div\Phi - \rho\Psi) dV = 0 \tag{11}$$

or finally

$$\dot{\rho} = div\Phi + \rho\Psi. \tag{12}$$

The last equation can be written for any part i_k of the information ensemble in partial form:

$$\dot{\rho}_i = div\Phi_i + \rho_i \Psi_i \tag{13}$$

[3] *Green's integral transformation using.*

Here, density, flow and production are correlated with part i_k of the information ensemble I.

Now dwell in more detail on the experimental study of information to energy transformation by M. Sano [2]. This work realized closed physical system which represents an isolated information ensemble of constant volume. There were no information flows in the experiment and all changes in the information ensemble were implemented due to negative internal production-reducing of information on a specified amount. The balance equation (integral form (10) is trivially reduced to a differential (12)) for this case can be written as:

$$\frac{1}{V}\frac{dI}{dt} = -\Psi \frac{I}{V} \tag{14}$$

which solution can be found as:

$$I = e^{-\Psi t}. \tag{15}$$

In the paper it was shown that information to energy transformation subordinated to Jarzynski equality:

$$e^{-\Delta F/kT} = e^{-W/kT}, \tag{16}$$

where according to the thermodynamics law,

$$\Delta F \leqslant W. \tag{17}$$

Here, ΔF – difference of the system free energy when work W is done (the equality in (17) occurs only for quasi-static process); k – Boltzmann constant; T – temperature.

Rewrite the left side of (16) as:

$$e^{-\Delta F/kT} = e^{-F_B/kT} \Big/ e^{-F_A/kT} \tag{18}$$

where F_A – free energy in the initial state and F_B – in the final state of the system; during the system transition from state A to B an amount of work W was done and free energy change on the value $\Delta F = F_B - F_A$. Note that in the experiment [2] during the transition from state A to B the information of the ensemble decreased for ΔI, so that

$$I_B \Big/ I_A = e^{-\Psi t_B} \Big/ e^{-\Psi t_A} = e^{-\Psi \Delta t} = e^{-\Delta I} \tag{19}$$

Comparing (16) and (19) we write the second thermodynamic law (17) for the case of information to energy transformation:

$$\frac{\Delta I}{I} \cdot kT = \Delta F \leqslant W. \tag{20}$$

Let us estimate the efficiency of information to energy transformation.

As we see from (16)–(20), it is important that the amount of ensemble information in the initial A and final states B only, then (related to famous efficiency estimation by Carnot–Clausius–Kelvin [3]):

$$e_I \leqslant 1 - I_B \Big/ I_A < 1. \tag{21}$$

or taking into account $\Delta I = I_A - I_B$:

$$e_I \leqslant \Delta I \Big/ I_A < 1. \tag{22}$$

We rewrite (22) in view of (20) as:

$$e_I \leqslant \frac{\Delta I}{I} = \frac{\Delta F}{kT} \leqslant \frac{W}{kT}\,. \tag{23}$$

Let us clarify the essence of the value ΔI: it is the part of complete ensemble information which can be realized as work and transformed to energy according to (20). In the experiment [2] ΔI – this is information about distribution of particles velocity in the physical system that has allowed transforming this information in a form of knowledge (about distribution of particles velocity) into work and energy according to (20). If we define information valuation in initial state A and final state B we obtain:

- in the initial state A, by knowing the velocity distribution of particles in the system, we with probability $p(\Delta I) \approx 1$ achieve the goal of information to energy transformation

$$J_A = \frac{p(\Delta I) - p(0)}{1 - p(0)} \approx 1; \tag{24}$$

- in the final state B, knowledge ΔI has lost its valuation because after transition from A to B and doing the work W the velocity distribution of particles in the system has changed and probability $p(\Delta I)$ getting re-work by initial knowledge is comparable to $p(0)$

$$J_B = \frac{p(\Delta I) - p(0)}{1 - p(0)} \approx 0 \leftarrow p(\Delta I) \cong p(0) \approx 0. \tag{25}$$

Here $p(0) \approx 0$ – the probability of achievement (getting a work) without knowledge about velocity distribution in the system. Thus, all information in the form of knowledge has been used for transformation into energy.

Transform the left side of (22) and consider the limiting case

$$\Delta I = e_I I. \tag{26}$$

Now we rewrite (26) according to Shannon's formula for the amount of information:

$$K(X) \equiv \Delta I = -e_I(X) \sum_m p(\mathrm{x}) \cdot \log p(\mathrm{x}), \quad e_I(X) \leqslant \frac{\Delta I}{I} \tag{27}$$

Thus, we statistically introduce the concept of **knowledge** $K(X)$ as the part

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Conclusion

Thus, the article introduced the basic axioms of the mathematical theory of information: from the elementary volume of information to the point of information and information ensemble of any size and complexity. It is shown that the current configuration of the information ensemble correlates with the contents of space, which is configured at the current time. It was present relations for the density of information at any point in the ensemble and noted that under certain assumptions, information density is determined only by the history of the information ensemble. It was written in the principle of conservation of information for open informational ensemble. The expression of the second law of thermodynamics for the transformation of information into energy was obtained based on the principle of conservation of information and data of the experiment M. Sano [3]. The assessment of the effectiveness of the information to energy transformation was done and it is shown that «knowledge» in simple physical system – is the part of the information that can be transformed into energy in accordance to the laws of thermodynamics. The probabilistic interpretation of the effectiveness of information to energy transformation was given, which introduces the value of knowledge through the formula for Shannon's information. Submitted relations are fundamental for the understanding and presentation of applied informatics sciences disciplines such us medical informatics.

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Математический подход к информации и знанию

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Резюме

В статье введена базовая аксиоматика математической теории информации: от элементарного информационного объема и точечной информации, до информационного ансамбля произвольного объема и сложности. Показано, что текущая конфигурация информационного ансамбля соотносит его содержание с пространством, которое конфигурация занимает в текущий момент времени. Представлены соотношения для плотности информации в любой точке ансамбля и отмечено, что при определенных предположениях плотность информации определяется только историей информационного ансамбля. Записан принцип сохранения информации для информационно открытого ансамбля и получено выражение второго закона термодинамики для случая трансформации информации в энергию. Приведена оценка эффективности трансформации информации в энергию и показано, что «знание» в простой физической системе — это часть информации, которая может быть трансформирована в энергию в соответствии с законами термодинамики. Дана вероятностная трактовка эффективности информационно-энергетической трансформации, которая вводит величину знания через формулу для информации Шеннона. Представленные соотношения являются базовыми для понимания и изложения прикладных дисциплин информатики, например, медицинской информатики.

Ключевые слова: информация, закон сохранения, термодинамика, знание.

Математичний підхід до інформації і знання

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Резюме

У статті введена базова аксіоматика математичної теорії інформації: від елементарного інформаційного обсягу та точкової інформації, до інформаційного ансамблю довільного обсягу та складності. Показано, що поточна конфігурація інформаційного ансамблю співвідносить його зміст з простором, який конфігурація займає в даний момент часу. Представлені співвідношення для щільності інформації в будь-якій точці ансамблю та зазначено, що при певних припущеннях щільність інформації визначається тільки історією інформаційного ансамблю. Записаний принцип збереження інформації для інформаційно відкритого ансамблю і отримано вираз другого закону термодинаміки для випадку трансформації інформації в енергію. Наведено оцінку ефективності трансформації інформації в енергію і показано, що «знання» в простій фізичної системі — це частина інформації, яка може бути трансформована в енергію відповідно до законів термодинаміки. Надано імовірнісне трактування ефективності інформаційно-енергетичної трансформації, яка вводить величину знання через формулу для інформації Шеннона. Представлені співвідношення є базовими для розуміння і викладання прикладних дисциплін інформатики, наприклад, медичної інформатики.

Ключові слова: інформація, закон збереження, термодинаміка, знання.

Переписка

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of information I that with probability $e_I(X)$ can be used for information to energy transformation according to (20)−(22).

Conclusion

Thus, the article introduced the basic axioms of the mathematical theory of information: from the elementary volume of information to the point of information and information ensemble of any size and complexity. It is shown that the current configuration of the information ensemble correlates with the contents of space, which is configured at the current time. It was present relations for the density of information at any point in the ensemble and noted that under certain assumptions, information density is determined only by the history of the information ensemble. It was written in the principle of conservation of information for open informational ensemble. The expression of the second law of thermodynamics for the transformation of information into energy was obtained based on the principle of conservation of information and data of the experiment M. Sano [3]. The assessment of the effectiveness of the information to energy transformation was done and it is shown that «knowledge» in simple physical system – is the part of the information that can be transformed into energy in accordance to the laws of thermodynamics. The probabilistic interpretation of the effectiveness of information to energy transformation was given, which introduces the value of knowledge through the formula for Shannon's information. Submitted relations are fundamental for the understanding and presentation of applied informatics sciences disciplines such us medical informatics.

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Математический подход к информации и знанию

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Резюме

В статье введена базовая аксиоматика математической теории информации: от элементарного информационного объема и точечной информации, до информационного ансамбля произвольного объема и сложности. Показано, что текущая конфигурация информационного ансамбля соотносит его содержание с пространством, которое конфигурация занимает в текущий момент времени. Представлены соотношения для плотности информации в любой точке ансамбля и отмечено, что при определенных предположениях плотность информации определяется только историей информационного ансамбля. Записан принцип сохранения информации для информационно открытого ансамбля и получено выражение второго закона термодинамики для случая трансформации информации в энергию. Приведена оценка эффективности трансформации информации в энергию и показано, что «знание» в простой физической системе — это часть информации, которая может быть трансформирована в энергию в соответствии с законами термодинамики. Дана вероятностная трактовка эффективности информационно-энергетической трансформации, которая вводит величину знания через формулу для информации Шеннона. Представленные соотношения являются базовыми для понимания и изложения прикладных дисциплин информатики, например, медицинской информатики.

Ключевые слова: информация, закон сохранения, термодинамика, знание.

Математичний підхід до інформації і знання

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Резюме

У статті введена базова аксіоматика математичної теорії інформації: від елементарного інформаційного обсягу та точкової інформації, до інформаційного ансамблю довільного обсягу та складності. Показано, що поточна конфігурація інформаційного ансамблю співвідносить його зміст з простором, який конфігурація займає в даний момент часу. Представлені співвідношення для щільності інформації в будь-якій точці ансамблю та зазначено, що при певних припущеннях щільність інформації визначається тільки історією інформаційного ансамблю. Записаний принцип збереження інформації для інформаційно відкритого ансамблю і отримано вираз другого закону термодинаміки для випадку трансформації інформації в енергію. Наведено оцінку ефективності трансформації інформації в енергію і показано, що «знання» в простій фізичній системі — це частина інформації, яка може бути трансформована в енергію відповідно до законів термодинаміки. Надано імовірнісне трактування ефективності інформаційно-енергетичної трансформації, яка вводить величину знання через формулу для інформації Шеннона. Представлені співвідношення є базовими для розуміння і викладання прикладних дисциплін інформатики, наприклад, медичної інформатики.

Ключові слова: інформація, закон збереження, термодинаміка, знання.

Переписка

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